Assignment 12

Exercises: 16.8 no 10, 14, 16, 17, 19, 20, 21.

No need to hand in any exercises.

Supplementary Problems

1. (Optional) Let Ω be a region in space which is bounded by a smooth closed surface S.

   (a) Use the divergence theorem to derive the formula of volume of Ω:

   $$|\Omega| = \frac{1}{3} \iint_S (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot \mathbf{n}\, d\sigma \ ,$$

   where $\mathbf{n}$ is the outer unit normal at S.

   (b) Assume that Ω is contained in a ball of radius R. Derive the inequality

   $$|\Omega| \leq \frac{1}{3}R|S| \ ,$$

   where $|S|$ is the surface area of S.

   (c) Find a region Ω so that the inequality in (b) becomes equality.